

VISVESWARAIAH TECHNOLOGICAL UNIVERSITY (VTU), BANGALORE (KARNATAKA)

Operations Research And Engineering Management

FOR

B.Tech (VIII Semester) Students of EC/TC/BM/ML
Courses of VTU and all other examinations.



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"Best Paper Award-2002", U.S.A.

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PREFACE

It gives me pleasure to inform you that the present edition on '**Operations Research and Engineering Management**' has been specially designed, made up-to-date and well-organised from the original book on '**Operations Research**' (*first published in 1972*) in a systematic order according to the latest syllabus as prescribed by VTU, Bangalore (Karnataka) for B. Tech (VIII Sem.) and all other students. It has grown out of the lectures given to the students during past 39 years. The entire book is divided into **Eleven** chapters. In this new edition :

- A large number of *solved examples* are given in each chapter.
- Sufficient *hints* and *answers* of all *examination problems* are given throughout the book.
- *Model Objective Questions* are given at the end of each chapter which are very useful for competitive examinations like IAS, IES etc.

In view of the above, it can be assured that this book will definitely raise the horizon of the knowledge in the subject of Operations Research.

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I am also thankful to the Publishers, Laser type setters Vibgyor and the Printers for its presentation in the present nice form.

Suggestions for further improvement of the book will be highly appreciated and thankfully acknowledged.

— S.D. SHARMA

D-215, Shastri Nagar
Meerut-250004 (U.P.) India

Dear Students,

May be your friends have recently appeared in any examination of *Operations Research*. I shall be highly grateful if you could please send me the question-papers and the syllabi of your course in this subject. The original papers will be returned soon after the use at your address.

—S.D. SHARMA

SYLLABUS

Visveswaraiiah Technological University (VTU), Bangalore (Karnataka)

OPERATIONS RESEARCH AND ENGINEERING MANAGEMENT

B. Tech (VIII Semester)

(EC/TC/BM/ML)

EC 71

-
1. **What is OR** : Operations Research model, Solving the OR model, Queueing and simulation models, Art of modeling, Phases of OR study. 02 Hrs.
 2. **Introduction to Linear Programming** : Two variable LP model, Graphical LP solution, Analysis of selected LP models. 03 Hrs
 3. **The Simplex Method** : LP solution space, Graphical to algebraic solution, The simplex method. Artificial starting solution, Special cases in simplex method applications. 05 Hrs
 4. **Transportation Model and Its Variants** : Definition of transportation model, Non-traditional transportation models, Transportation algorithms, Assignment model. 05 Hrs
 5. **Network Models** : Network definitions, Minimal spanning tree algorithm, CPM and PERT. 05 Hrs
 6. **Game Theory** : Optimal solution of two persons zero sum games, Solution of mixed strategy games. 04 Hrs
 7. **Queueing Systems** : Why study queues ? Elements of queueing model, Pure-birth and death model, Generalized queueing model, Single server models. 03 Hrs
 8. **Introduction to Engineering Management** : Engineering and Management Historical Development of Engineering Management 04 Hrs

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COMPUTER PROGRAMMS IN 'C'

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